IE 361 Module 25

Introduction to Shewhart Control Charting Part 2 (Statistical Process Control, or More Helpfully: Statistical Process Monitoring)

Reading: Section 3.1 *Statistical Methods for Quality Assurance*

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The population of numbers on washers used in the "Deming Drama" in IE 361 Labs is (approximately) Normal with mean $\mu = 5$ and standard deviation $\sigma = 1.715$. If (as in the Deming Drama) one is sampling $n = 5$ washers per period and doing process monitoring with the hope of maintaining the "brown bag parameters," appropriate control limits for $\overline{x}$ are

$$UCL_{\overline{x}} = 5 + 3\frac{1.715}{\sqrt{5}} = 7.3 \text{ and } LCL_{\overline{x}} = 5 - 3\frac{1.715}{\sqrt{5}} = 2.7$$

and these are to be applied in an "on-line" fashion to sample means as they are observed.

The table on the next panel shows sample means and ranges for 15 samples collected in a Deming Drama. The first 12 of those samples were drawn from the brown bag and the last 3 from a much different population.

Standards Given Charting for Means

Example 25-1 continued

Sample	1	2	3	4	5	6	7	8	9
$\overline{x}$	5.4	6.2	6.0	5.6	4.4	3.4	4.0	4.8	4.4
R	3	5	5	6	6	5	2	2	3

Sample	10	11	12	13	14	15
$\overline{x}$	6.2	5.6	4.8	8.0	9.6	12.8
R	2	6	4	10	9	5

Note that all 12 of the samples actually drawn from the brown bag have sample means inside the control limits for $\overline{x}$. Only after the process change between samples 12 and 13 (indicated by the vertical double line in the table) do the $\overline{x}$'s begin to plot outside the standards given control limits 2.7 and 7.3.

Retrospective Charting for Means

Example 25-1 continued

But what could one do in the way of control charting if the brown bag parameters were not provided? Is it possible to look at the 13 sample means and ranges in the table above and conclude that there was clearly some kind of process change over the period of data collection? One might reason (retrospectively) as follows.

IF in fact there had been no process change, all 15 of the sample means $\overline{x}$ would be estimates of μ (that we know to be 5 for the first 12 samples). So a reasonable empirical replacement for μ in control limit formulas might be

$$\overline{\overline{x}} = \frac{5.4 + 6.2 + \cdots + 12.8}{15} = 6.08$$

Further (in a manner similar to what we did in the range-based estimation of $\sigma_{\text{repeatability}}$ in Gauge R&R analyses), one might invent an estimate of σ from each of the 15 ranges in the table, as $\hat{\sigma} = R/d_2$, where d_2 is based on the sample size of $n = 5$.

Supposing again that there had been no process change in the data collection, these might be averaged across the 15 samples to produce

$$\frac{\bar{R}}{d_2} = \frac{(3+5+\cdots+9+5)\,/15}{2.326} = 2.092$$

as a potential empirical replacement for σ in control limit formulas. That is, as-past-data control limits for $\overline{x}$ in the Deming Drama might have been

$$CL_{\overline{x}} = \hat{\mu} = 6.08$$

$$\begin{aligned} UCL_{\overline{x}} &= \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \\ &= 6.08 + 3\frac{2.092}{\sqrt{5}} = 8.9 \end{aligned}$$

and

$$LCL_{\overline{x}} = 6.08 - 3\frac{2.092}{\sqrt{5}} = 3.3$$

Retrospective Charting for Means

Example 25-1 continued

If the retrospective limits on panel 5 are applied to the 15 sample means in the table on panel 3, we see that (although the change between samples 12 and 13 is not caught until sample 14) even though there are 3 samples from a process different from the brown bag process that "contaminate" our estimates of the brown bag parameters, this method of retrospective calculation does detect the fact that there was some kind of process change in the period of data collection.

Rational Subgrouping

A matter that needs some discussion is that of **rational subgrouping** or rational sampling. It is essential that what one calls a "sample" be collected over a period short enough that there is little question that the process was "physically stable" during that period. A "random draws from a fixed distribution" model must be appropriate for describing data in a given "sample." This is because the variation within such a sample essentially specifies the level of "background noise" against which one looks for process change. If what one calls "samples" often contain data from genuinely different process conditions, the level of background noise will be so large as to mask important process changes.

In high-volume manufacturing applications, single samples (rational subgroups) typically consist of n consecutive items taken from a production line. On the other hand, in extremely low-volume (or batch) operations, where one unit of product might take many hours to produce and there is significant opportunity for real process change between consecutive units, the only natural samples may be of size $n = 1$.

Control Limits and Specifications

An unfortunate and persistent type of confusion for students (and even experienced engineers) concerns the *much different* concepts of control limits and engineering specifications.

Control Limits	Specifications
have to do with process stability	have to do with product acceptability
apply to Q	apply to individuals, x
usually derive from process data	derive from performance requirements

A real process can be stable without being acceptable and vice versa. There is *not* a direct link between being "in control" and producing acceptable results.

The Deming Drama as typically run in an IE 361 Lab uses the understanding that washers from $L = 3$ to $U = 7$ are "in specifications" /functional, while those outside those limits are not. This has nothing whatsoever to do with either the truth about the brown bag process or any data collected in the Drama. In fact, using the process parameters one might compute $z_1 = (U - \mu)/\sigma = (7-5)/1.715 = 1.17$ and $z_2 = (L - \mu)/\sigma = (3-5)/1.715 = -1.17$ and note that the fact that

$$P\left[-1.17 \leq \text{a standard normal random variable} \leq 1.17\right] \approx .76$$

suggests that only about 76% of the brown bag process actually meets those specifications/requirements. (We are here ignoring the fact that the basic discreteness of what is drawn from the brown bag serves to make this estimate lower than the truth.) And, of course, lacking known values for μ and σ, one could use past data to estimate μ and σ and then this kind of *fraction conforming* for the brown bag process.

What Control Charting Can and Cannot Do

Control charting can signal the need for process intervention and can keep one from ill-advised and detrimental over-adjustment of a process that is behaving in a stable fashion. But in doing so, what is achieved is simply reducing variation to the minimum possible for a given system configuration (in terms of equipment, methods of operation, methods of measurement, etc.). That achieving of process stability provides a solid background against which to evaluate possible innovations and fundamental/order-of-magnitude improvements in production methods.