IE 361 Module 55

Design and Analysis of Experiments Part 15
(Some Perspective and a Very-Small-Fraction Example)

Reading: Section 6.1 *Statistical Methods for Quality Assurance*

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Some Perspective

We reiterate some perspective gained in the last module and the analysis of the 1/4 fraction of Hanson and Best: The larger is q, the larger the inevitable ambiguity of interpretation of 2^{p-q} fractional factorial results, and the more likely the need for follow-up study. *Small fractions are really most useful as screening studies, to pick a few likely candidates out of many potentially important factors for subsequent more detailed study.*

We end with an extreme example involving a large q, i.e. a **small fraction**.

Example 50-1 continued

The Hendrix chemical process study involved $p = 15$ factors A, B, C, D, E, F, G, H, J, K, L, M, N, O, P (the factor names and levels were given earlier).

Here $p - q = 4$, i.e., only $2^4 = 16$ combinations were run !!!!! This was a $\frac{1}{2^{15-4}} = \frac{1}{2048}$ fraction !!!!

The 11 generators used were:

$$\begin{aligned} \text{E} &\leftrightarrow \text{ABCD,F} \leftrightarrow \text{BCD,G} \leftrightarrow \text{ACD,H} \leftrightarrow \text{ABC,J} \leftrightarrow \text{ABD,} \\ \text{K} &\leftrightarrow \text{CD,L} \leftrightarrow \text{BD,M} \leftrightarrow \text{AD,N} \leftrightarrow \text{BC,O} \leftrightarrow \text{AC,P} \leftrightarrow \text{AB} \end{aligned}$$

These led to the 16 combinations and (ultimately) the data on panel 4.

Example 50-1 continued

combination	y
eklmnop	14.8
aghjkln	16.3
bfhjkmo	23.5
abefgkp	23.9
cfghlmp	19.6
acefjlo	18.6
bcegjmn	22.3
abchnop	22.2
dfgjnop	17.8
adefhmn	18.9
bdeghlo	23.1
abdjlmp	21.8
cdehjkp	16.6
acdgkmo	16.7
bcdfkln	23.5
abcdefghjklmnop	24.9

Figure: Hendrix 2^{15-11} Data

Example 23-1 continued

Pretty clearly it isn't sensible to write out the whole defining relation here. Effects are going to be aliased in 16 groups of $2^{11} = 2048$ effects. But for a most tentative interpretation, let's see what we might glean if the physical system is so simple that only main effects dominate. (Whether this is physically reasonable is a question that needs to be answered by a process expert.)

The 16 observations are listed in Yates order for factors A,B,C and D (ignoring the rest). We therefore begin by running them through the Yates algorithm, with the results on panel 6.

Example 50-1 continued

combination	y	estimate (16 divisor)	sum estimated
eklmnop	14.8	20.28	grand mean+ $\cdots$
aghjkln	16.3	.13	A+ $\cdots$
bfhjkmo	23.5	2.87	B+ $\cdots$
abefgkp	23.9	−.08	AB+P+ $\cdots$
cfghlmp	19.6	.27	C+ $\cdots$
acefjlo	18.6	−.08	AC+O+ $\cdots$
bcegjmn	22.3	−.19	BC+N+ $\cdots$
abchnop	22.2	.36	ABC+H+ $\cdots$
dfgjnop	17.8	.13	D+ $\cdots$
adefhmn	18.9	.03	AD+M+ $\cdots$
bdeghlo	23.1	.04	BD+L+ $\cdots$
abdjlmp	21.8	−.06	ABD+J+ $\cdots$
cdehjkp	16.6	−.26	CD+K+ $\cdots$
acdgkmo	16.7	.29	ACD+G+ $\cdots$
bcdfkln	23.5	1.06	BCD+F+ $\cdots$
abcdefghjklmnop	24.9	.11	ABCD+E+ $\cdots$

Figure: Result of the Yates Algorithm for the Hendrix Data

Example 50-1 continued

There is no replication in this data set, so we're driven to normal plotting in order to judge statistical significance of these estimates. A normal plot of the (last 15) estimates is below.

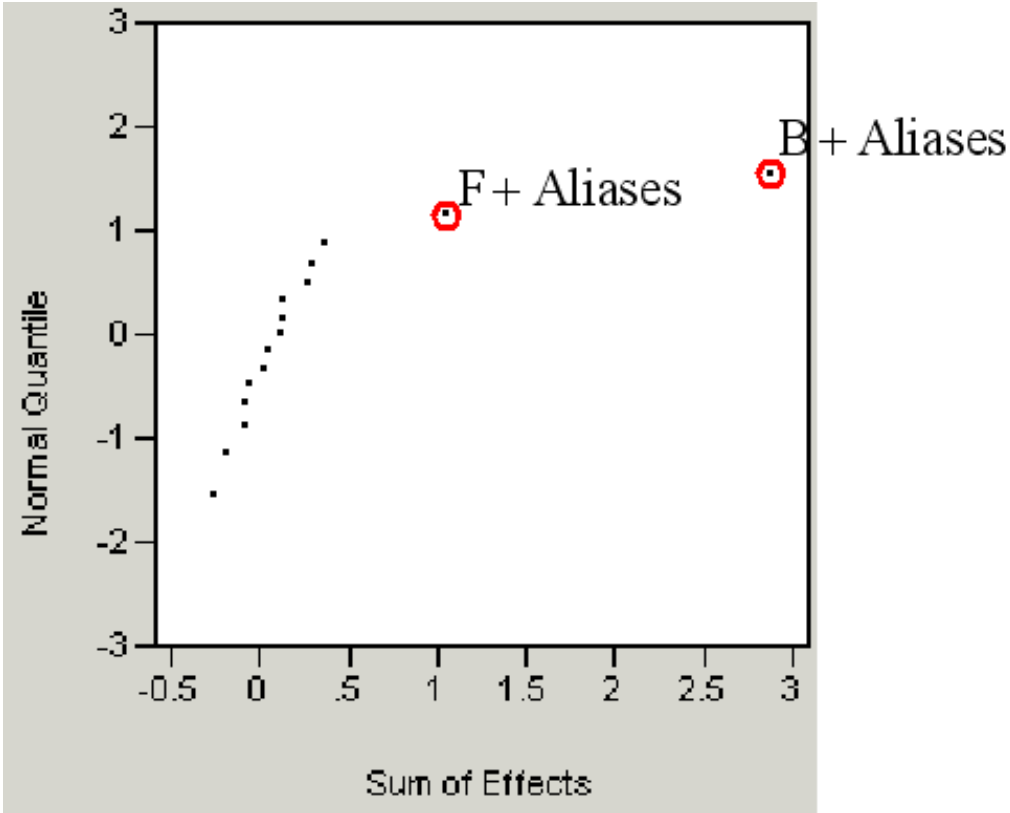

Figure: Normal Plot of 15 Estimated Sums of Effects from the 2^{15-11} Chemical Process Study

Example 50-1 continued

The plot shows that there are two fitted sums of effects that are clearly statistically detectable. As each of these stands for a sum of 2048 effects, no solid final conclusions can be made. But if one assumes that the "big" sums are primarily driven by the main effects appearing in them, a plausible tentative interpretation is that the most important factors appear to be

B (Solvent) and F (# of Chill Rolls)

and for large cold crack resistance "high B" (refined solvent) and "high F" (2 chill rolls) appear best.

Note that the analysis does point out what is in retrospect quite obvious, namely that it is those combinations in the data set with "high B" and "high F" that have the largest y's.