SVMs Part 3D: Perspective

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Toy SVM example

The "kernelizing" of the support vector classifier methodology produces a wide variety of possible classifiers that can be tuned (via cross-validation) over choice of kernel (and any parameters it might have) and C^* or C. As a p=1 and N=20 example of what can result from the technology, consider the situation (based on kernels $\mathcal{K}\left(x,z\right)=\exp\left(-\gamma\left(x-z\right)^2\right)$) portrayed on the next slide.

Pictured are voting functions that are approximations to the optimal 0-1 loss classifier as linear combinations of the N=20 radial basis functions $\exp\left(-\gamma\left(x-x_i\right)^2\right)$ plus a constant. The $\gamma=100$ pictures are understandably more wiggly than the $\gamma=10$ pictures because of the smaller "bandwidth" of the former basis functions. The $C^*=1000$ pictures are closer to being the "hard margin" situation and have fewer training case errors in evidence.

Voting functions

Here are SVM voting functions. Red bars pointing up from the "rug" are for xs with y=-1 and blue are for xs with y=1. Black bars point down for xs corresponding to "support vectors."

$$\gamma = 10$$

$$\gamma = 100$$

$$C^* = 1000$$

$$C^* = .1$$

Some perspective

Remember in all this, that SVMs built on a kernel \mathcal{K} will choose voting functions that are linear combinations of the functions $\mathcal{K}\left(\mathbf{x}_{i},\cdot\right)$, slices of the kernel at training case inputs. That fact controls what "shapes" are possible for those voting functions. (In this regard, note that the kernel defined by the ordinary Euclidean inner product, $\mathcal{K}\left(\mathbf{x},\mathbf{z}\right)=\langle\mathbf{x},\mathbf{z}\rangle$, produces linear voting functions and thus linear decision boundaries in \Re^{p} and the special case of ordinary support vector classifiers. It is sometimes called the "linear kernel.")

It is important to keep in mind that to the extent that SVMs produce good voting functions, those must be equivalent to approximate likelihood ratios. The discussion of Section 1.5.1 still stands!

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